Launch Vehicle Acoustics Part 2: Statistics of the Time Domain Data

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Skewness and kurtosis coefficients were investigated as a means of quantifying the shock content and non-Gaussian characteristics of rocket noise. Rocket noise data measured at 3-5 locations during the launches of four different vehicles with thrusts ranging from 440 to 10,700 kN were analyzed. The kurtosis coefficients of the acoustic pressure data showed no discernible pattern of variation about the Gaussian value of 3. The skewness coefficients ranged from 0.02 to 0.55, all of them greater than the value of 0 expected for Gaussian data. Both the skewness and kurtosis coefficients for the derivative of the acoustic pressure showed much greater deviations from the values expected for Gaussian noise. This study has confirmed that the coefficient of skewness is a useful metric for the characterization of rocket noise. The statistics of the pressure gradient have been shown to be more sensitive indicators of shock content, and these metrics are much less sensitive to low-frequency instrumentation limits.

Nomenclature

 D_e = equivalent engine exit diameter, m

d = distance from launch pad to observer, m

f = frequency, Hz

 f_c = center frequency, Hz P = acoustic pressure, Pa

 \dot{P} = gradient of the acoustic pressure, Pa/s

 x_i = value of the *i*th digitized data point

 μ = mean value, nominally zero

 σ = standard deviation or rms amplitude

 $\langle \rangle$ = time average

Introduction

RECENTLY, a series of acoustic measurements was made during the launches of four different vehicles: a Peace-keeper, a Delta, a Scout, and a Titan IV. The thrust of these vehicles ranged from the Scout's 440 kN to the Titan IV's 10,700 kN. Acoustic data were recorded at distances of 0.30, 0.61, and 1.22 km from the Peacekeeper pad; 0.46, 0.61, and 0.92 km from the Delta pad; 0.36, 0.64, 0.94, and 1.22 km from the Scout pad; and 0.82, 2.04, 3.41, 5.79, and 13.15 km from the Titan IV pad. Data were measured using sound level meters (SLMs) and the data recorded on digital audio tape (DAT) recorders. Details of the instrumentation system are given in Ref. 1.

The spectral data in Ref. 1 indicated possible nonlinear propagation effects. There was an apparent lack of high-frequency absorption when overall sound pressure levels (OASPLs, re 20×10^{-6} Pa) averaged over the 6-dB down period were greater than 120 dB, i.e., the 6-dB down spectra did not display the expected atmospheric absorption at high frequencies. Even at locations where the average (over the 6-dB down period) OASPL was less than 120 dB, the high-frequency absorption was much less than that expected on the basis of the absorption factors applicable to linear acoustic levels.

High level rocket noise is rich in shock content and is positively skewed. Since the high radiation efficiency of rockets is attributed to Mach wave radiation from the plume, it is not

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surprising that rocket noise data contain shock fronts. Shocks have also been measured in supersonic jet noise by Ffowcs-Williams et al.² and by Laufer et al.³ It was noted in Ref. 2 that shock fronts in jet noise data (perceived as crackle) are not well characterized in the spectral domain. In this article, statistics of the time domain data are given that do characterize this shock content. Measured waveforms and statistical parameters calculated from the recorded time series data are presented. The influence of the low-frequency response of the instrumentation system on the characteristics of the measured data is discussed. The data are shown to be strongly non-Gaussian, the derivative of the time domain data having skewness and kurtosis coefficients that deviate much more strongly from Gaussian values than do those of the data itself.

Influence of Low-Frequency Instrumentation Response

Ffowcs-Williams et al.² found no correlation between spectral representations of jet noise and its crackle component and proposed skewness as an appropriate statistical measure for quantifying crackle. They discussed the effect of low-frequency roll-off, dc, and near dc-coupling in the instrumentation system, and stated that "when the expansion between jumps is too slow for the equipment to handle faithfully," the recording process could induce skewness. On the basis of later experiments, they concluded that their recorded signals had been faithful reproductions of the originals.

A study by Geoff et al.⁴ demonstrated the influence of lowand high-frequency instrumentation limits on the metrics used to characterize blast waves. Geoff et al. simulated different low-frequency instrumentation roll-offs by high-pass filtering their recorded blast wave data at 1.0, 10, and 20 Hz (presumably 3-dB down points). The phase responses of the high-pass filters were not given. Because the blast waveforms studied in Ref. 4 are very similar to the shock waveforms found in highintensity rocket noise, it was hoped that that study might provide some insight into the influence of the instrumentation system on the statistical parameters calculated in this study. A detailed review of the findings of Ref. 4 did not, however, provide any definitive indication of the effect that the instrumentation system might be expected to have on the data reported here.

The greatest concern, here, is the effect of the low-frequency instrumentation response on the Titan IV data sets. The low-frequency cutoff (3-dB down point) of the measurement sys-

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tems used to record the data analyzed in this study is estimated to be between 5–10 Hz (Ref. 1). Peak levels in the Titan IV sound power spectral densities (PSDs), calculated from the recorded data and averaged over the 6-dB down period, occur at a frequency of 16 Hz. On average, the measured Titan IV PSD levels are down only 1.5 dB from the peak at 10 Hz. Peak frequencies in the Peacekeeper, Delta, and Scout PSDs are significantly higher, ranging from 35 to 55 Hz. In these PSDs, levels are down 5–9 dB from the peak at 10 Hz.

Reference 5 reports the findings of an investigation into the influence of low-frequency instrumentation roll-off and phase distortion on rocket noise metrics. The baseline data set in that study was Taurus launch noise measured using a carrier microphone system, which has excellent low-frequency measurement capabilities. (The Taurus vehicle uses a modified Peacekeeper engine, so that the intensity and frequency distribution of the Taurus launch noise is comparable to that of the Peacekeeper.) Low-frequency instrumentation system responses were simulated using high-pass filters, following the approach of Ref. 4. The Taurus data were time scaled to simulate noise with the same frequency distribution as the Titan IV.

The key finding of Ref. 5 was that the low-frequency distortion induced by the instrumentation system can be expected to increase the skewness of the measured Titan IV data. The corresponding influence on the average OASPL (over the 6-dB down period) and on other statistics of the Titan IV time domain data was negligible. The same (simulated) low-frequency instrumentation response resulted in only a small increase in the skewness of the Taurus data. Notably, it was the phase distortion at low frequencies, as opposed to the roll-off in response magnitude, that had the greatest influence on the skewness of the data.

Analysis of Recorded Rocket Noise Data

The majority of the data analysis summarized in this article was performed on a Unix workstation using PV-Wave Advantage, a data visualization and analysis program. Specialized procedures written for the analysis of launch vehicle and satellite data were utilized. Some of the statistical parameters considered in the analysis are discussed in the following.

6-dB Down Average OASPLs and Crest Factors

OASPLs were averaged over the period during which the 1-s averaged OASPL was within 6 dB of the maximum (the 6-dB down period). It was found, in Ref. 1, that when the 6-dB down levels exceeded 120 dB, the normalized high-frequency levels showed an apparent absence of high-frequency absorption. A possible explanation for this is the existence of nonlinear propagation effects. Because peak amplitudes, as opposed to rms levels, may provide a better indication of the potential for nonlinearity, a crest factor was determined for each data set (but not the derivative data sets):

crest factor =
$$20 \log_{10}(\text{maximum/rms}) dB$$
 (1)

The maximum instantaneous sound pressures and rms sound pressures were those over the 6-dB down time period.

Coefficient of Skewness

The third and fourth statistical moments, skewness and kurtosis, respectively, can be used to assess whether or not a random variable has a normal distribution.⁶ The skewness coefficient is given by

skewness coefficient =
$$\sum_{i=1}^{n} \frac{(x_i - \mu)^3}{\sigma^3}$$
 (2)

Here, the summation i = 1, n encompasses the number of instantaneous data values in the 6-dB down averaging period. The data were digitized at a rate of 48,000 samples/s.

In the case of Gaussian data, the skewness coefficient is zero. A positive skewness indicates a greater probability of large positive values and a nonsymmetric probability density distribution. That is, a positive skewness coefficient indicates high-sided data, where positive pressure maxima are larger in magnitude than the negative pressure minima.

Coefficient of Kurtosis

There are two commonly used definitions of the coefficient of kurtosis. The definition utilized in this study is given in Eq. (3). Using this definition, the value of the coefficient of kurtosis for a normally distributed variable is 3:

kurtosis coefficient =
$$\sum_{i=1}^{n} \frac{(x_i - \mu)^4}{\sigma^4}$$
 (3)

Derivative of Time Domain Data

Root mean square values, coefficients of skewness and coefficients of kurtosis were also calculated for the derivatives (numerically evaluated) of the digital time domain data. The derivative of a Gaussian variable is also Gaussian, since differentiation is a linear operation. The statistics of the derivative were calculated, because very early in the measurement program it became apparent that the derivative was a more sensitive indicator of the shock content of the data. Consider, for example, the time domain plots in Fig. 1. The plot of pressure vs time in Fig. 1a shows the characteristic high sidedness, or skewness, of high-intensity rocket noise, but (because of the compressed time scale), it does not suggest the associated shock content. However, the pressure gradient time history (Fig. 1b), readily reveals the shock content of the data; it is the steep positive slopes of the shock waveforms that are re-

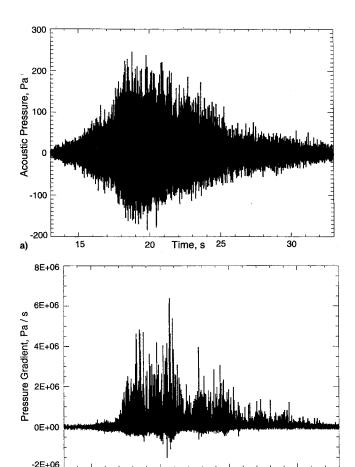


Fig. 1 Delta launch noise measured 0.46 km from the pad: a) acoustic pressure and b) acoustic pressure gradient.

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15

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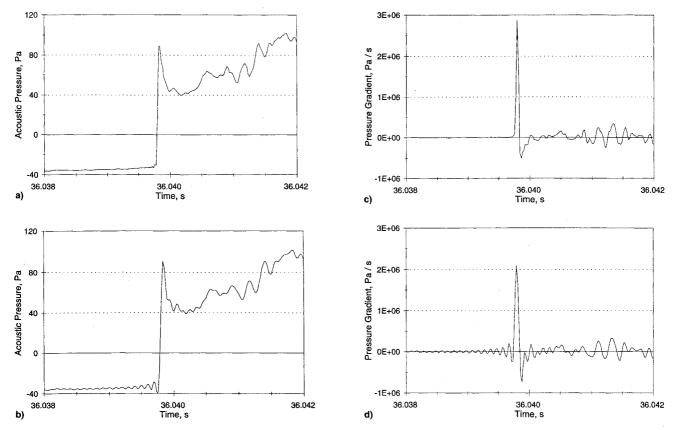


Fig. 2 Effects of low-pass filter on shock waveform. Filter was a 10-kHz low-pass FIR filter (120 filter coefficients, Gibbs factor of 1, flat phase over 0- to 10-kHz range): a) as recorded, b) low-pass filtered, c) derivative of waveform in a, and d) derivative of waveform in b.

sponsible for the large positive values of the derivative. Derivative time series were derived from the acoustic pressure data using a standard PV-wave advantage procedure that utilizes three-point Lagrange interpolation. The statistics of the pressure gradient are based on the instantaneous values over the 6-dB down period.

Digital Low-Pass Filtering

Concern with the potential for noise amplification by the differentiation process, coupled with the fact that the one-third-octave band spectra for the two most distant Titan IV locations dropped into the noise floor at frequencies above 10 kHz, led to the decision to low-pass filter the data prior to differentiation. For simplicity and consistency, all of the launch data sets were identically low-pass filtered. The digital filter applied was a finite impulse response (FIR) low-pass filter with a cutoff frequency of 10 kHz, 120 filter coefficients, and a Gibbs factor of 1. The filter phase was flat over the 0- to 10-kHz range.

There is reason to believe that low-pass filtering of the data was not necessary. The results show that low-pass filtering affected the magnitude of the statistical measures of the derivative data sets. Figures 2a and 2b show a shock front in the Titan IV noise measured at 3.4 km from the launch pad before and after low-pass filtering, respectively. The oscillations in front of the shock are a result of the sharp filter roll-off, but are of little consequence in the calculated statistical averages. The influence of the low-pass filter on the slope of the shock front is not easy to see in Figs. 2a and 2b, but is starkly apparent when the corresponding maximum derivative values are compared (Figs. 2c and 2d). Note that for Figs. 2c and 2d the derivative was calculated using forward differencing.

It must be stated, then, that the average 6-dB down statistics for the derivatives are not considered quantitatively accurate; though low-pass filtering did eliminate uncertainties associated with variations in the high-frequency roll-off of the instrumentation systems. In any case, the trend and the order of magnitude of the statistics of the derivatives are of significant interest.

Effective Center Frequency

For a given vehicle, the standard deviation of the pressure gradient decreases with increasing measurement distance more rapidly than does the rms pressure. This is consistent with high-frequency losses because of atmospheric absorption. The rms derivative and rms sound pressure can be used to define an effective f_c :

$$rms \dot{P} = (2\pi f_c) * rms P \tag{4}$$

Atmospheric absorption acts to reduce the effective center frequency of the noise spectrum.

Averaging Period

Initially, statistical parameters were calculated for the time domain data and for the derivative time series using 1- and, then, 2-s time averages. Plots of some of these short duration statistics are presented in the results. However, the information generated by the use of small averaging periods proved unmanageable, and the results were difficult to compare. It was decided to concentrate on averages calculated over the entire 6-dB down period.

As the distance from the source increases, peak sound pressure levels decrease and the lobe of maximum directivity takes longer to sweep over a fixed observer location. Consequently, averaging times vary from location-to-location (increasing with increasing distance), and vehicle-to-vehicle (as the 6-dB down time also depends on vehicle ascent rate). For the data measured during the three smaller vehicle launches, the 6-dB down time period (determined from 1-s averaged OASPLs) ranged from 7.5 to 16.6 s. For the Titan IV data sets, averaging times ranged from 10.8 to 36.5 s.

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Table 1 Launch noise statistics over the 6-dB down time period*

d, km	d/D₄	OASPL, dB	Crest factor, dB	Skewness of P	Kurtosis of P	rms <i>Þ</i> , kPa/s	Skewness of P	Kurtosis of P	f₅, Hz
Delta									
0.46	400	128.7	13.0	0.35	3.1	150	7.1	100	420
0.61	530	128.3	12.4	0.071	2.9	100	7.7	110	315
0.92	800	122.8	14.8	0.041	3.0	61	7.7	120	350
Scout									
0.36	420	125.7	13.2	0.12	3.2	100 (110)	6.9 (9.1)	100 (180)	425
0.64	740	118.9	14.8	0.14	3.3	38	3.8 (4.4)	77 (100)	340
0.94	1100	115.2	15.8	0.021	3.2	17 (18)	5.5 (6.8)	160 (280)	240
1.22	1410	111.5	14.7	0.063 (0.042)	3.1	11 (8)	3.6 (3.3)	49 (41)	220
Peacekeeper				` ,		` '	` ,	` '	
0.30	210	134.9	>12.9b	0.21 ^b	3.2	210	7.7	120	295
0.61	420	128.2	12.8	0.13	3.0	98	9.6	180	300
1.22	840	123.3	13.9	0.43	3.3	37	8.3	150	205
Titan IV									
0.82	245	139.5	14.4	0.55	3.5	240 (250)	9.8 (12)	200 (310)	203
2.04	605	129.0	11.3	0.42 (0.44)	2.9 (3.0)	50	17 (22)	620 (980)	140
3.41	1020	127.0	13.0	0.38 (0.36)	2.8	24 (25)	13 (19)	700 (150ó)	85
5.79	1725	119.0	12.6	0.074 (0.070)	2.9	11 (7.4)	8.0 (7.8)	250 (240)	75
13.15	3920	109.2	15.5	0.32	3.0 (2.9)	1.3 (1.4)	1.7 (1.6)	9.9 (9.8)	40

^aNumbers in parentheses represent values for data analyses without low-pass filtering.

Results and Discussion

The values of the 6-dB down statistical averages for the four launch vehicles are given in Table 1. For two of the vehicles, the Scout and Titan IV, the values calculated from the data before low-pass filtering are given in parentheses (no values are quoted if the unfiltered and filtered data produced the same statistical values). A comparison of the gradient skewness and kurtosis coefficients for the filtered and unfiltered derivative time series (Scout and Titan IV) substantiates the effect of the low-pass filter illustrated in Fig. 2. The decrease in skewness and kurtosis coefficients that resulted from low-pass filtering at 10 kHz is less pronounced at the distant sites, where the high-frequency energy content was already reduced by atmospheric absorption. Only for the data recorded at the most distant Scout location and at the two most distant Titan IV locations are the values of the unfiltered data statistics closer to the Gaussian values than those of the filtered data. This is consistent with the fact that the (unfiltered) sound pressures measured at these locations were dominated by background noise at frequencies above 10 kHz.

Using the rms \dot{P} values from Table 1 and the corresponding rms P value calculated from the OASPL, the effective center frequencies [see Eq. (4)] for the 6-dB down data sets were estimated. These estimates are given in the last column of Table 1. In Ref. 1, plots of normalized PSDs were presented for each of the vehicles. The normalized spectra corresponding to sites where the 6-dB down OASPLs exceeded 120 dB showed little or no apparent change in the high-frequency spectral distribution. However, the general decrease in f_c with increasing distance shown in Table 1 indicates that high-frequency losses were, of course, incurred even when the 6-dB down OASPLs exceeded 120 dB.

Statistics of Acoustic Pressure Time Series

Deviations of the kurtosis of the acoustic pressure data sets from the Gaussian value of 3 (see Table 1) are both positive and negative, but all of the values are close to 3.0. Surprisingly, there is not a correlation between larger deviations from the Gaussian value and large skewness coefficients. The kurtosis coefficient of the acoustic pressure does not appear to be a good indicator of the unique characteristics of this data.

The skewness coefficients in Table 1 are all greater than zero, ranging in value from 0.02 to 0.55. All of these values are believed to be somewhat overstated, particularly those of the Titan IV data sets, as a result of the low-frequency instrumentation response.⁵ It was anticipated that the shocks in

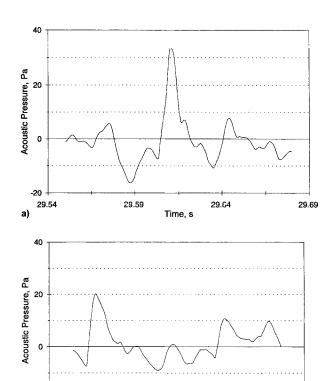


Fig. 3 Typical waveforms in the Titan IV launch noise measured 13.15 km from the pad: during period when 2-s averaged skewness was close to a) 0.9 and b) 0.25.

38.31

38.36

38.26

38.21

rocket noise data would be reflected in positive skewness values. Given that atmospheric absorption and spherical spreading act to reduce the intensity of the noise and the shock strengths, it was also expected that skewness values would decrease with distance from the source. For a given vehicle, the sound pressure skewness values do tend to decrease with distance, but this trend is reversed at the most distant Scout, Peacekeeper, and Titan IV measurement locations.

A possible explanation for the increase in skewness values at the most distant Scout, Peacekeeper, and Titan IV locations, relative to the next closer measurement location, can be found

^bFive positive pressure peaks were clipped at the input to the DAT recorder.

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in the sound pressure spectral distributions. The frequency response curves, both phase and magnitude, measured for the SLMs (exclusive of microphones) at the most distant and next closer locations were very nearly identical for these measurements. Nonetheless, the normalized spectral levels for the Peacekeeper and Scout data at the most distant locations were significantly lower at frequencies below the peak in the spectrum (see Ref. 1). The larger skewness values at the more distant sites are, then, associated with the sound pressure spectra that are relatively deficient in energy at frequencies below the peak in the spectrum. This is consistent with the findings of Ref. 5, except that in this case the reduction in low-frequency levels is not attributed to the instrumentation system.

In the case of the Titan IV data, the distribution of energy at frequencies below the peak is similar in the spectra at 5.8 and 13.15 km (Ref. 1), but the effective center frequency of the data measured 13.15 km is only 40 Hz, whereas in the noise measured at 5.8 km it is 75 Hz. As might be expected, this suggests that the low-frequency distortion introduced by the instrumentation system (i.e., the increase in skewness) becomes more significant as the effective center frequency of the data is reduced. Typical waveforms in the Titan IV launch noise measured 13.15 km from the pad are shown in Fig. 3. Only vestiges of shock waveforms can be seen in this noise data.

Statistics of the Derivative Time Series

The skewness and kurtosis coefficients for the derivative data sets both show substantial deviations from the Gaussian values. The size of these deviations is much greater than those of the pressure data itself, consistent with the substantial difference in character between the data and their derivative, as illustrated in Fig. 1. The skewness and kurtosis values of the pressure gradient tend to increase and then decrease again with increasing distance from the source, but the data of Table 1 are not sufficient to establish a definitive trend. Analyses similar to those reported in Ref. 5 indicate that the statistics of the

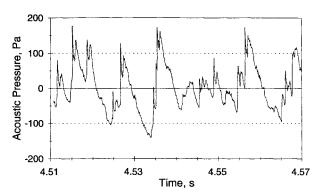


Fig. 4 Typical waveforms in the Scout launch noise measured 0.36 km from the pad.

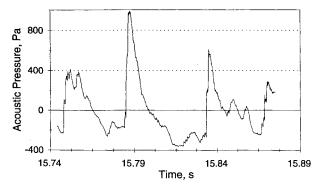


Fig. 5 Typical waveforms in the Titan IV launch noise measured 0.82 km from the pad.

pressure gradient data were not affected by the low-frequency instrumentation response.

The skewness and kurtosis values for the Titan IV pressure gradient data sets are large relative to those of the Delta, Scout, and Peacekeeper. The explanation for this is not readily apparent, but there are two significant differences between the Titan IV and the other data sets. First, the peak frequency of the Titan IV launch noise is substantially lower than that of

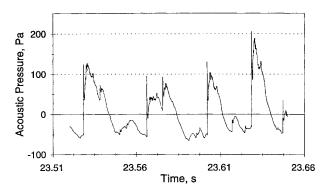


Fig. 6 Typical waveforms in the Titan IV launch noise measured 2.04 km from the pad.

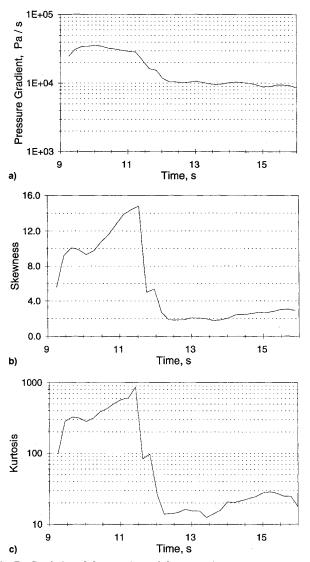


Fig. 7 Statistics of the gradient of the acoustic pressure measured 0.94 km from the Scout pad. Plots are sliding 2-s averages: a) standard deviation, b) skewness of gradient, and c) kurtosis of gradient.

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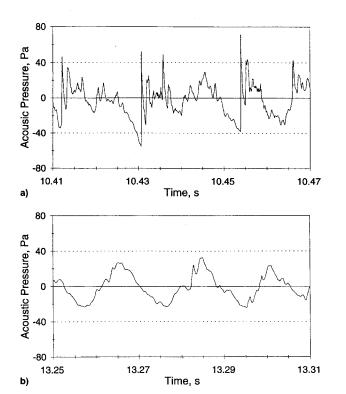


Fig. 8 Waveforms in the Scout launch noise measured 0.94 km from the pad. During period of a) large and b) small gradient skewness and kurtosis.

smaller vehicles. Secondly, for equal OASPLs, the Titan IV sets were measured considerably further from the source, increasing the potential for atmospheric effects such as scattering by turbulence. Scattering by atmospheric turbulence has been suggested as an explanation for the narrow, positive pressure spikes measured in some sonic boom waveforms.⁷

Short Duration Averages

Plots of sliding 2-s averages of the OASPL, skewness, and kurtosis of the acoustic pressure time series and of the standard deviation (rms value), skewness, and kurtosis of the derivative time series were generated for all of the data sets. In the case of the Delta, Scout, and Peacekeeper locations (all of which were located within 1.25 km of the pads), these plots showed fairly smooth to little variation over the 6-dB down time period. The only exceptions to this were statistics of the pressure gradient at the Scout locations 0.94 and 1.22 km from the pad. Figure 4 is a typical segment of Scout data measured 0.36 km from the pad during the 6-dB down time period.

For the Titan IV data measured at the two closest locations, the 2-s averaged skewness and kurtosis coefficients of the pressure gradient were also uniformly high during the 6-dB down period. Figure 5 is a typical segment of Titan IV data recorded at the closest location, 0.82 km from the pad, and Fig. 6 is a sample of the Titan IV data measured 2.04 km from the pad. The derivative skewness and kurtosis coefficients for the Titan IV data measured 13.15 km from the pad were uniformly low across the 6-dB down period, reflecting the absence of steep shock fronts and consistent with the reduction in high-frequency spectral levels (see Fig. 3).

The skewness and kurtosis of the pressure gradient at the Scout locations 0.94 and 1.22 km from the pad showed substantial variation within the 6-dB down period. Plots of the 2-

s average standard deviation, skewness, and kurtosis coefficients for the pressure gradient 0.94 km from the Scout pad are shown in Fig. 7. To verify that high-level skewness and kurtosis coefficients for the pressure gradient were correlated with the presence of shocks in the acoustic pressure data, the corresponding time domain data sets were closely examined. Two plots of waveforms taken from the Scout data set corresponding to the statistics in Fig. 7 are shown in Fig. 8. The first waveform in Fig. 8a, displaying shock fronts and spikes, was measured during the period when the 2-s averaged skewness and kurtosis coefficients for the derivative (Figs. 7b and 7c, respectively), were large. The second waveform (Fig. 8b) was measured later in the 6-dB down time period, when the derivative skewness and kurtosis coefficients were smaller. The waveforms in Fig. 8 can be compared to those in Fig. 4 measured 0.36 km from the Scout pad.

Conclusions

The skewness coefficients for the acoustic pressure data ranged from 0.02 to 0.55, all greater than the zero value for Gaussian data. The kurtosis coefficients for the same data did not display a discernible pattern of variation about the Gaussian value of 3. Both the skewness and kurtosis coefficients for the derivative of the acoustic pressure showed much greater deviations from the values expected for Gaussian noise. It has been established that high-level rocket noise is non-Gaussian and that the coefficient of skewness is a useful indicator of the shock content of the noise. However, the statistics of the pressure gradient appear to be more sensitive indicators of the shock content of the data than are the statistics of the data itself, and these metrics are less sensitive to low-frequency instrumentation limitations.

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